Mathematics Specialist Unit 2

Test 5 –Trig Equations & Number Proofs

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Weight: 6% Overall Total: \_\_\_\_\_\_\_\_

## Section One: Calculator Free 25 minutes 24 marks

**Show working out where appropriate.**

**Question 1 [6 marks]**

The function has been graphed below.



a) Determine the values of the constants *a, b* and *c*. [3 marks]

b) On the same set of axes, sketch the graph of . [2 marks]

c) State the number of solutions to the equation over the domain .

[1 mark]

**Question 2 [7 marks]**

Find all the solutions to the following equations over the given domain.

a) ,  [2 marks]

b) ,  [2 marks]

c) ,  [3 marks]

**Question 3 [3 marks]**

Determine the exact solutions for the following equation:

 for .

**Question 4 [4 marks]**

A trigonometric function has equation . Determine the values of *a* (where *a* > 0) and *b* given that *P* has a maximum value of 4 when .

**Question 5 [4 marks]**

Given , prove that if is odd then *x* is even.

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## Section Two: Calculator Assumed 35 minutes 34 marks

**Calculators allowed plus a page of notes.**

**Show working out where appropriate. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.**

**Question 6 [3 marks]**

By using trigonometric identities and without the use solely of a calculator, solve for all values of *x* (radians) for .

**Question 7 [3 marks]**

Use an appropriate trigonometric technique to find in exact form the maximum value of  and the values of *x* at which this occurs.

**Question 8 [12 marks]**

a) Consider the expression .

i) Write down the values of for *m* = 1, 3, 5, 7 and 9. [1 mark]

ii) Use your values from i) to state the largest integer,, that  is always divisible by, when is a positive odd integer. [1 mark]

iii) Prove that is always divisible by  when  is a positive odd integer. [4 marks]

b) Let .

i) Show that  when  [1 mark]

ii) Prove by the principle of mathematical induction that  is divisible by 7.

[5 marks]

**Question 9 [7 marks]**

The motion of a small body moving along a straight track was recorded by a video camera for 20 seconds. An analysis of the motion showed that the distance, *x* cm, of the body from a fixed point *O* on its path *t* seconds after recording began was given by .

a) The distance can also be given by , where *a* and *b* are real constants. Determine the values of *a* and *b*. [2 marks]

b) Graph on the axes below for . [3 marks]



c) State the period and amplitude of the graph of . [2 marks]

**Question 10 [6 marks]**

The graphs of the functions  are shown below, where

*a, b, c, d, e* and *f* are real constants.



State the values of constants *a, b, c, d, e* and *f*.

**Question 11 [3 marks]**

Prove that  is a rational number.